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Galois modularity as foundation of highly efficient exact algorithms in Classical Mechanics

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Evariste Galois' last letter (written on the eve of his murder May 30, 1832) [1], eloquently described by Hermann Weyl as "the most substantial piece of writing in the whole literature of mankind", contained explicit constructions for depressing the degree of the modular equation of (prime) levels 5, 7 and 11. In particular, the (Galois) group the modular equation of level 5 (which degree is depressable from 6 to 5) coincides with the group of the (general) quintic. Nonetheless, and while Galois' contribution to formulating necessary and sufficient condition for solving algebraic equations via radicals is widely known, Galois' decisive contribution to actually solving the quintic is barely (if at all) recognized. And, in fact, Galois must be solely and entirely credited for bringing to light the tight intertwinement of calculating the roots of the modular equation of level p with calculating the p-torsin points on a corresponding elliptic curve [2,3,4].

Even the most basic problems of dynamics such are concerning the motion of a simple pendulum or a free rigid body [5,6,7], or a static problem, concerning tether equilibria in linear parallel force field [8,9], cannot be exhaustively solved without determining the (full) structure of the group of transformations, which carry out a solution to another. And the construction of highly efficient exact algorithms in Classical Mechanics necessarily rely upon highly efficient arithmetic on elliptic curves, as well as, efficient calculation of complete elliptic integrals of all (three) kinds [10].

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