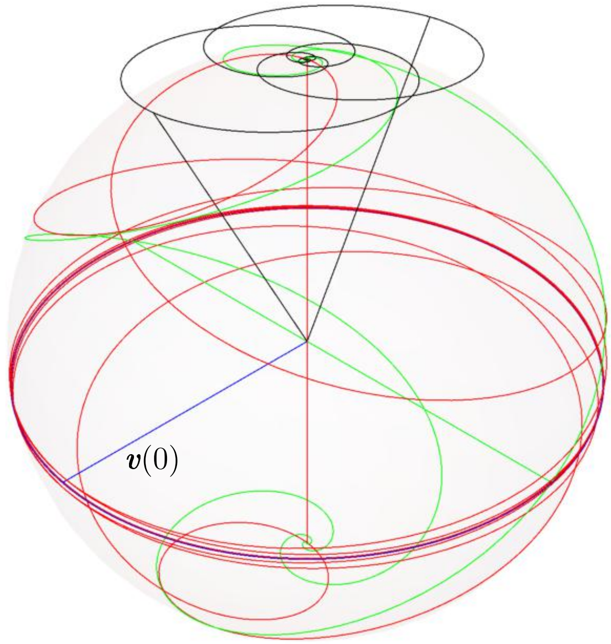
Dzhanibekov's flipping wingnut and Burke's twisting tennis racket

Semjon Adlaj (Russia, Moscow)

CC FRC "Informatics and Control" of RAS

SemjonAdlaj@gmail.com

The fundamental problem of torque free rigid body motion has traditionally bewildered researchers throughout the globe [1]. A widely acclaimed geometric description of such motion via a rolling (without slipping) Poinsôt ellipsoid turns out being elusive in "critical" cases, evidenced by Vladimir Dzhanibekov and demonstrated by William Burke. As is the case with the simple pendulum "standing" in its unstable equilibrium position which "separates" two rotary motions (in two distinct directions): "clockwise" and "counterclockwise" [2], a free rigid body spinning about its middle axis of inertia might "flip" in two distinct "oppositely oriented" ways! With the point at (complex) infinity being added to the time domain, of the critical solution, the uniqueness (which, otherwise, holds for a solution restricted to a bounded time domain) is violated. And, as was the case with the pendulum where two motion regimens (oscillatory and rotary) separated by unstable equilibrium must be distinguished, two motion regimens for a freely moving rigid body separated by "permanent" rotation about the middle axis must also be distinguished. Namely, a rotation might occur about either the "minor" or the "major" axis but it never occurs (simultaneously) about both. An analytic unifying solution of free rigid body motion, explicitly expressed via time dependent transition matrices, requires a (complete) determination of the group of its "preserving" fractional transformations, as was pointed out in [3]. We shall discover that achieving an exact and computationally robust solution requires the construction of a fourth axis (along with body's three main axes of inertia), which we call Galois critical axis. It (and only it) rotates uniformly and permanently about the (fixed) angular momentum, even as the middle axis "reverses" its direction (to either match or oppose the direction of the angular momentum) during the critical motion which must (from now on) necessarily "augment" the permanent rotation about the middle axis, routinely characterized as "unstable" (mistakenly) suggesting that no other solutions emerge unless perturbations (however small) ensue. Moreover, dual critical solutions sharing one and the same (invariant) Galois critical axis must be "analytically continued at (complex) infinity" before we declare the motion completely determined and the problem entirely settled!



The trajectories for two dual solutions in a "König reference frame". Both solutions share one and the same Galois critical axis $\boldsymbol{v}$, which is shown in blue (at $t = 0$). The (green) trajectory of the tip of the middle axis of inertia "begins" at one "pole" and "ends" at the other. The directions of the middle axes, corresponding to dual solutions, are opposed to each other at the "equator", spanned by the (blue) tip of Galois critical axis. Further clarification is given in [4].

References

[1] Maxwell C.J. *On a Dynamical Top, for exhibiting the phenomena of the motion of a system of invariable form about a fixed point, with some suggestions as to the Earth's motion* // Transactions of the Royal Society of Edinburgh, **XXI** (IV), Read on April 20$^{\text{th}}$, 1857: 559-570.

[2] Adlaj S. *An analytic unifying formula of oscillatory and rotary motion of a simple pendulum* (dedicated to 70$^{\text{th}}$ birthday of Jan Jerzy Slawianowski) // Proceedings of International Conference "Geometry, Integrability, Mechanics and Quantization", Varna, Bulgaria, 2014, June 6-11: 160-171.

[3] Adlaj S. *Dzhanibekov's flipping nut and Feynman's wobbling plate* // Polynomial Computer Algebra International Conference, St. Petersburg, Russia, 2016, April 18-23: 10-14.

[4] Adlaj S. *Torque free motion of a rigid body: from Feynman wobbling plate to Dzhanibekov flipping wingnut*. Available at `http://www.ccas.ru/depart/mechanics/TUMUS/Adlaj/FRBM.pdf`.