A 29-gon construction via 3 and 7 angle section

Semjon Adlaj

Let $\xi := e^{2\pi\sqrt{-1}/7}$. The construction of a primitive 7^{th} root of unity via (no more than) cube root extraction was explicitly shown here. Introduce the four (complex) numbers

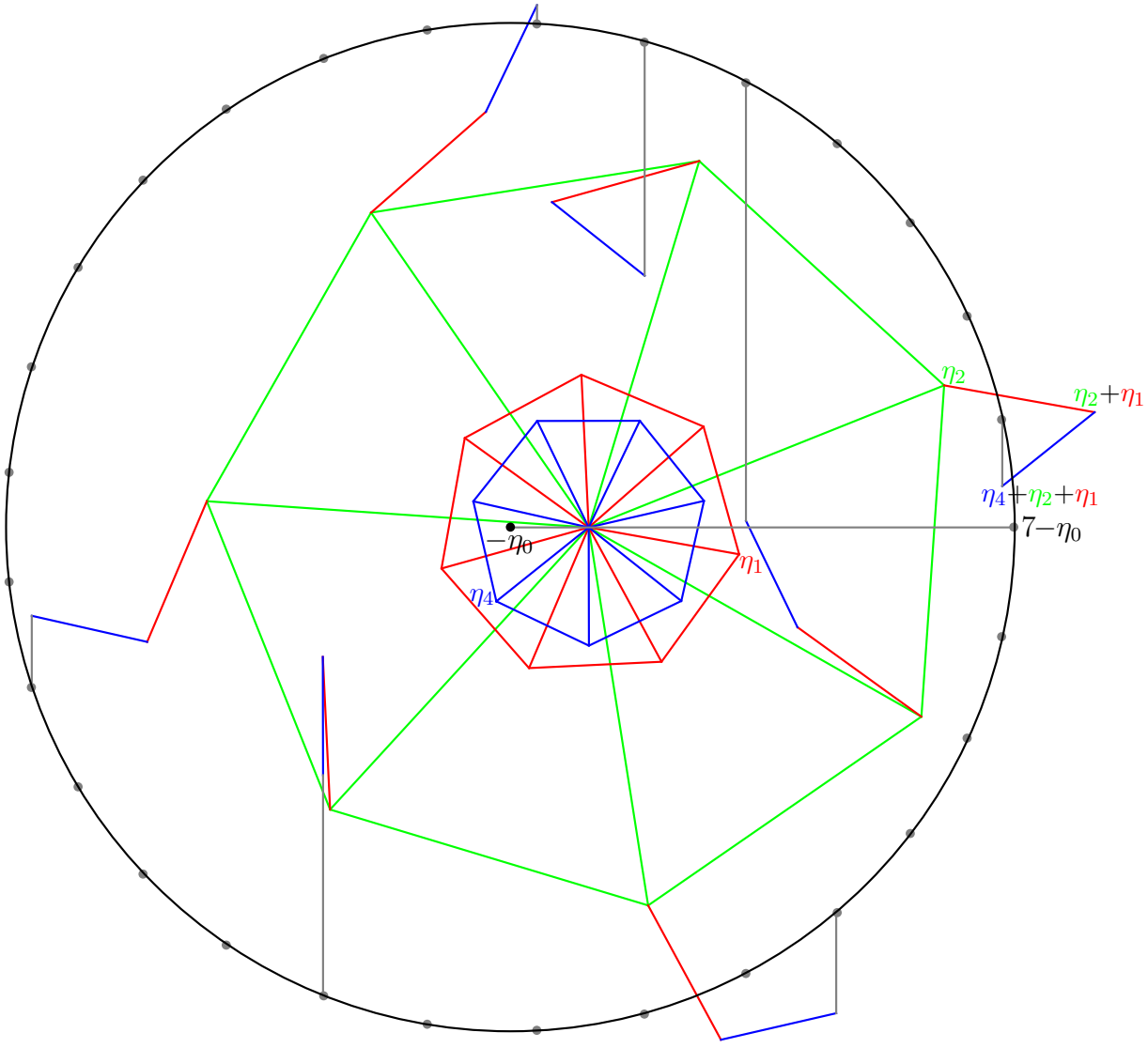
$$\eta_0 := \frac{\sqrt{29}-1}{4},\ \eta_1^7 := p(\xi),\ \eta_2^7 := p(\xi^2),\ \eta_4^7 := p(\xi^4),$$

where

$$p(x) := \frac{7(18879-1709\sqrt{29}-6641\sqrt{-7}-2237\sqrt{-203}\,)}{4}\,x^2+$$

$$+\frac{7(19981-6411\sqrt{29}-8845\sqrt{-7}+439\sqrt{-203}\,)}{4}\,x-\frac{84071}{2}-8557\sqrt{29}-10962\sqrt{-7}+\frac{4445\sqrt{-203}}{2}.$$

The construction of the icosienneagon which we propose assumes that we have extracted the 7^{th} roots η_1, η_2 and η_4 which correspond to particular vertices of three (colored in red, green and blue) heptagons.



Construct a circle, centered at $-\eta_0$, with radius 7, and place a vertex on it at $(29-\sqrt{29})/4$. Then the real part of the sum $\eta_1+\eta_2+\eta_4$ turns out to match the real part of a "next" vertex of a 29-gon thereby inscribed in the just-constructed circle.