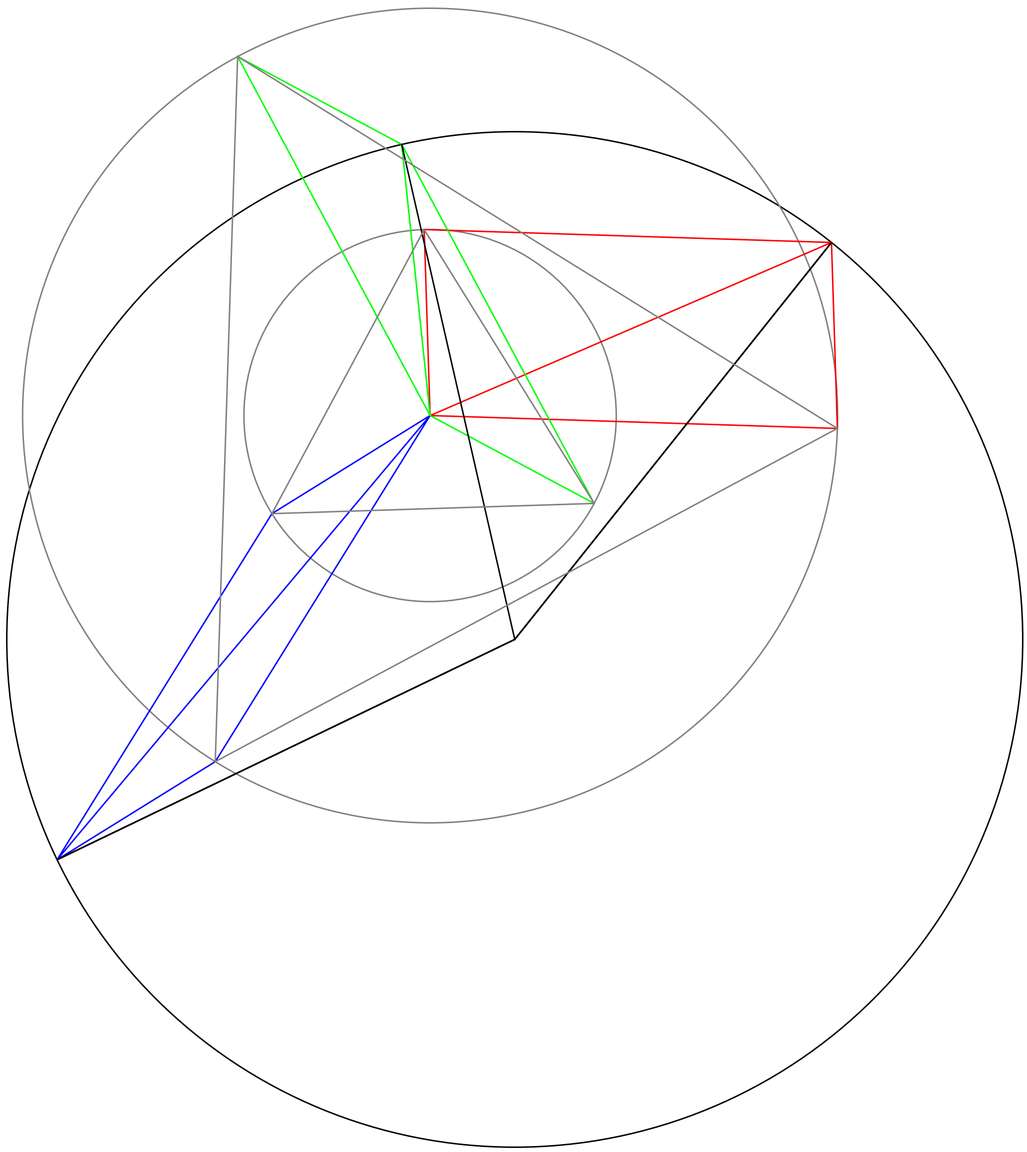
The heptagon constructed!

by Semjon Adlaj

The construction, being offered, requires trisecting the angle $\theta := \arctan\left(3\sqrt{3}\right)$. Once this is done, we draw two concentric (grey) circles with radii $\sqrt{\left(7 \pm \sqrt{21}\right)/18}$. We shall refer to their common center as *the center* and we shall assume that all subsequent rotations are performed about it.



Either one of the two (gray) inscribed triangles might be "aligned" with the other if rotated (counter-clockwise or clockwise, respectively) by the angle $\theta/3$.[1] Construct a (blue) parallelogram from three vertices which are, along with the center, a (selected) vertex on the larger triangle and a (selected) vertex on the smaller triangle. Two other (green and red) parallelograms possess a common vertex (at the center). Two other vertices of the green (or red) parallelogram are obtained by rotating, by the angle $2\pi/3$, counter-clockwise (or clockwise) the vertex of the blue parallelogram on the larger triangle, and rotating in the opposite direction the vertex (of the blue parallelogram) on the smaller triangle. The three formed vertices of the three constructed parallelograms diagonally opposing their common vertex (at the center) constitute three vertices of a heptagon, which turns out to be inscribed in a unit (black) circle, which center lies at a distance $\sqrt{2}/3$ from the center. Moreover, the vertices of the heptagon might be ordered as the first (red), second (green) and fourth (blue) vertex of the heptagon. They correspond to the three quadratic residues modulo the prime 7.

[1]Note that no ambiguity arises even when the angle $\theta/3$ is defined (merely) up to $2\pi/3$, since no vertices have been yet selected.