A hendecagon construction via quintisection

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The construction, as offered, requires angle quintisections. So, assume that we are given the ten vertices of two regular pentagons. The vertices are the fifth roots of the two (complex) numbers ζ_+^5 and ζ_-^5, where

$$\zeta_\pm^5 := \frac{\pm 25\sqrt{5} - 89 - 5\,i\,\sqrt{410 \pm 178\,\sqrt{5}}}{44\sqrt{11}},\ i := \sqrt{-1}.$$

Note that both numbers ζ_+^5 and ζ_-^5 lie in the third quadrant, and let ζ_+ and ζ_- denote their corresponding fifth roots in the first quadrant. Construct a circle centered at $1/\sqrt{44}$ with radius $5/\sqrt{11}$, and place a vertex on it at $\sqrt{11}/2$. Then the real part of the sum $\zeta_- + \zeta_+$ turns out to match the real part of a "next" vertex of a hendecagon thereby inscribed in the just-constructed circle.