A heiskaitriacontagon construction via quintisection

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Introduce the (three) complex numbers

$$\eta_0 := \frac{\sqrt[3]{31\left(2+3\sqrt{-3}\right)}+\sqrt[3]{31\left(2-3\sqrt{-3}\right)}-1}{6},\ \eta_1^5 := p(\xi),\ \eta_2^5 := p(\xi^2),$$

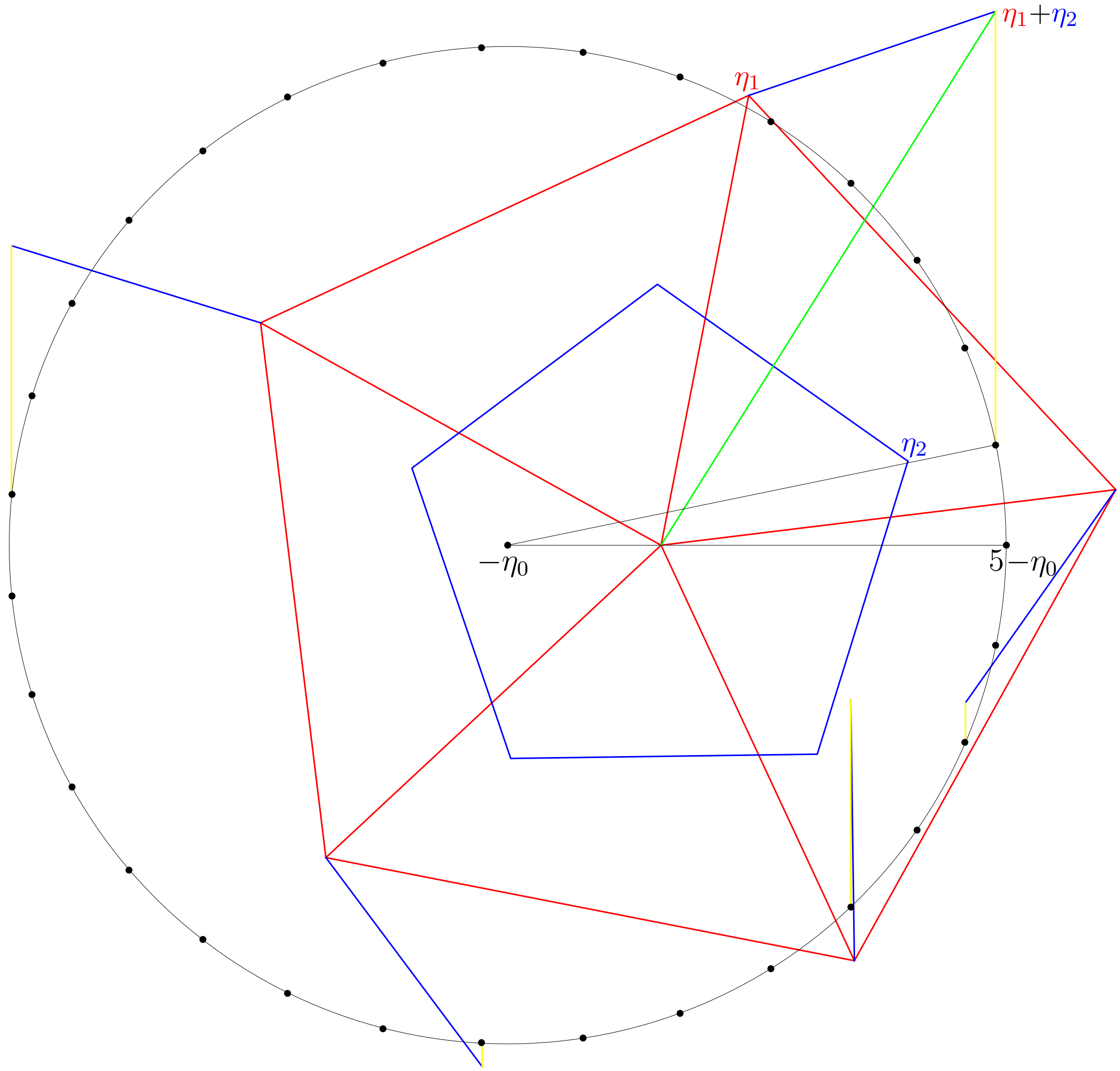
where

$$\xi := e^{\frac{2\pi\sqrt{-1}}{5}} = \frac{\sqrt{-2(5+\sqrt{5})}+\sqrt{5}-1}{4},\ p(x) := \frac{\sqrt[3]{31\left(2+3\sqrt{-3}\right)}\,z_+(x)+\sqrt[3]{31\left(2-3\sqrt{-3}\right)}\,z_-(x)-z_0(x)}{6},$$

$$z_0(x) := 280\,x^4+2140\,x^3-30\,x^2-960\,x-1208,$$

$$z_\pm(x) := -50\,x^4-215\,x^3-60\,x^2+255\,x+274\pm\sqrt{-3}\left(60\,x^4-5\,x^3+130\,x^2-115\,x-44\right).$$

The construction of the heiskaitriacontagon which we propose assumes that we have extracted the 5^{th} roots η_1 and η_2 which correspond to particular vertices of two (colored in red and blue) pentagons.



Construct a circle, centered at $-\eta_0$, with radius 5, and place a vertex on it at

$$5-\eta_0 = \frac{31-\sqrt[3]{31\left(2+3\sqrt{-3}\right)}-\sqrt[3]{31\left(2-3\sqrt{-3}\right)}}{6}.$$

Then the real part of the sum $\eta_1+\eta_2$ turns out to match the real part of a "next" vertex of a 31-gon thereby inscribed in the just-constructed circle.