

Two problems posed and solved by Alexandre Borovik

Semjon Adlaj

On September 19, 2024 Alexandre Borovik presented his “Ideas for a radical reform of mathematics education, après Arnold, Gelfand, and Vavilov”. There, he posed two problems.

The Gulnar Average Score Problem

Ideas for a radical reform of mathematics education, après Arnold, Gelfand, and Vavilov
└ Some cognitive aspects: Abstraction, encapsulation, reification

Encapsulation / de-encapsulation and Khan Academy

Gulnar has an average score of 87 after 6 tests. What does Gulnar need to get on the next test to finish with an average of 78 on all 7 tests?

Hints provided, one after another, by the Khan Academy website:

Hint 1. Since the average score of the first 6 tests is 87, the sum of the scores of the first 6 tests is $6 \times 87 = 522$.

Hint 2. If Gulnar gets a score of x on the 7th test, then the average score on all 7 tests will be:

$$\frac{522 + x}{7}.$$

Hint 3. This average needs to be equal to 78 so:

$$\frac{522 + x}{7} = 78.$$

Hint 4 $x = 24$.

“Gulnar has an average score of 87 after 6 tests. What does Gulnar need to get on the next test to finish with an average of 78 on all 7 tests?”

Once the problem was posed, we were told (without being asked) of “hints provided, one after another, by the Khan Academy website”. We immediately found ourselves thrown off track with the very first (and most irrelevant) “hint”. Therefore, we must pose and ask: why on Earth do we need to “sum the scores of the first 6 tests”? Instead of explaining the purpose of this summation, we were then introduced to the “Questions circa method”, according to which “students themselves” are expected to first ask “how many points in total did Gulnar get in 6 tests?”

Once again, we must object to the expectations of this imposed question and reiterate our own (which Alexandre Borovik fails to expect): why repeatedly sum the scores without considering the unnecessary of this arithmetic action for solving the problem?

Let us explain that only a single multiplication of two digits, namely $9 (= 87 - 78)$ and 7 (the total number of tests), is required for solving the said problem. Subtracting the product ($63 = 9 \cdot 7$) from 87 gives us the desired answer: 24 .

Students should be told that our calculation does NOT require calculating either of the two (terribly unnecessary products), namely 522 and 546 , which Alexandre Borovik calculates. Imposing needless calculations upon students is a great disservice to mathematical education, which Nikolai Vavilov wholeheartedly opposed.

The Cruising in the Mississippi River Steamboat Problem

Ideas for a radical reform of mathematics education, après Arnold, Gelfand, and Vavilov
 ↳ Some cognitive aspects: Abstraction, encapsulation, reification

Reification in arithmetic: intermediate parameters
It takes 5 days for a steamboat to get from St Louis to New Orleans, and 7 days to return back. How long will it take for a raft to drift from St Louis to New Orleans? ↓

Let us **introduce** a new unit of distance, *lieue* and **set** the distance from StL to NO being equal to $5 \times 7 = 35$ lieue.
 Speeds of the steamboat:

$$\text{downstream } \frac{35 \text{ lieue}}{5 \text{ day}} = 7 \frac{\text{lieue}}{\text{day}}, \text{ upstream } \frac{35 \text{ lieue}}{7 \text{ day}} = 5 \frac{\text{lieue}}{\text{day}}.$$

Since the speed of the current gets added to, or subtracted from, the speed of the steamship in still water, the speed of the current is

$$\frac{7 \text{ lieue/day} - 5 \text{ lieue/day}}{2} = 1 \frac{\text{lieue}}{\text{day}}$$

and a raft will drift for $35 \text{ lieue} / 1 \text{ lieue/day} = 35$ days.

“It takes 5 days for a steamboat to get from St Louis to New Orleans, and 7 days to return back. How long will it take for a raft to drift from St Louis to New Orleans?”¹

Instead of observing that the time required for the steamboat to travel the same distance in still water is the harmonic mean of the two given times (for traveling back and forth),² Alexandre Borovik calculates the “speed of the current” which is neither required nor meaningful in itself.³ He fails to recognize a compelling example of an inversion, that is, a function that coincides with its inverse. In other words, the ratio of the lesser given time to the greater given time is inverted to the ratio of the time required for the steamboat to travel in still water to the time of the current.⁴

Mathematics is a wonderful human endeavor, and Nikolai Vavilov did his best to convey that the mathematical education ought not be reduced to formal (and thoughtless) procedures.

¹Later, the “steamboat” evolved into a “steamship” in Alexandre Borovik’s solution.

²In particular, the harmonic mean of 5 and 7 is $35/6$.

³Just as knowing the distance (in lieues or any other units) between Saint Louis and New Orleans is not required for solving this problem

⁴That is, the inversion $x \mapsto (1 - x)/(1 + x)$ carries the ratio $5/7$ to the ratio $1/6$ (and vice versa).