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Tether equilibria in proximity to a circularly orbiting satellite and their stability criteria

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Impose an orthogonal orbital coordinate system $Oxyz$, whose origin O is assumed to be circularly orbiting, with the Oz-axis directed outwards along the radius vector, and the Ox-axis pointing in the direction of the orbital motion along its tangent. The potential u (due to gravitational and centrifugal forces), if linearized, is independent of the x-coordinate [1] and is then given by the formula

$$u(x, y, z) = \frac{\omega^2}{2}\Big(y^2 - 3z^2\Big),$$

where ω is the (assumed constant) angular speed of the orbital motion.

The forms of equilibria of an absolutely flexible nonstretchable homogeneous tether, whose endpoints (x_0, y_0, z_0) and (x_l, y_l, z_l) are fixed in the orbital coordinate system, are defined as the conditional extremals of the functional

$$U = \int_0^l \left(y^2 - 3z^2\right) \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, dt,$$

$$(x(0), y(0), z(0)) = (x_0, y_0, z_0), \ (x(l), y(l), z(l)) = (x_l, y_l, z_l),$$

subject to a constraint equation (corresponding to a prescribed tether length), where the dots denote differentiation with respect to a parameter t which might eventually be assumed natural [2].

Aside from planar solutions in three mutually orthogonal planes (the plane of the orbit, the plane orthogonal to the radius vector, and a plane orthogonal to the direction of the orbital motion), the equilibrium curve generally possesses a torsion vanishing at a discrete set of points, and the tangent to this spatial curve at any of these zero torsion points necessarily intersects the Ox-axis (without exceptions need to be made, as long as the Ox-axis is augmented with the point at infinity).

A key for classifying forms of equilibria, for a linearized force field in a neighborhood of a circularly orbiting satellite, is an investigation into the three special afore indicated planar cases, the first two of which might be modeled by a linear parallel force field, where the solutions are given analytically as graphs of elliptic and trigonometric functions [3]. Pojaritsky [4] proved that only a single equilibrium form, from a countably infinite family of equilibria, described by Appell [5], of a tether, subject to a repelling force field whose magnitude is linearly proportional to the distance from an axis to which the ends of the tether are being attached, is stable. Here, we shall proceed to geometrically constructing a function assigning to any given point on such an equilibrium curve its conjugate point.

In the general case, when the equilibrium curve is spatial possessing only a discrete set of zero torsion points, we show that, the strengthened Legendre condition is satisfied, and hence, the strengthened Jacobi condition is sufficient for guaranteeing stability.

References

[1] Beletsky, V. V., Levin, E. M.: Dynamics of Space Tether Systems. American Astronautical Society, San Diego, Calif. (1993).

[2] Forsyth, A. R.: Calculus of variations. Cambridge University Press, London (1927).

[3] Adlaj, S. F.: Tether equilibria in a linear parallel force field // 4th International Young Researchers Workshop on Geometry, Mechanics and Control, Ghent, Belgium, January 11-13, 2010. http://www.wgmc.ugent.be/adlaj.pdf (23 pages).

[4] Pojaritsky, G. K.: Stability of equilibria of mechanical systems, including flexible nonstretchable tether // Prikladnaya Matematika i Mekhanika, vol. 37, 4 (1973) 647–658 (in Russian).

[5] Appell, P.: Traite de Mecanique Rationnelle. T. 1. Gauthier-Villars, Paris (1924).