

The arc length of the sine curve

The arc length of a half period of the sine curve is efficiently calculated as $M + \pi/M$, where M is the arithmetic-geometric mean of 1 and $\sqrt{2}$, and the reciprocal of M is known as Gauss constant. See [footnote 7 of my article](#) “An eloquent formula for the perimeter of an ellipse”.

The graph of the sine function might be viewed as a graph of a “degenerate” elliptic function. Generally, the length of a period of the graph of an elliptic function is expressed via [the modified arithmetic-geometric mean](#) which was, in fact, introduced for efficiently calculating the length of a thread in a linear parallel force field. See [page 54 of my monograph](#) “Равновесие нити в линейном параллельном поле сил: Классификация и исследование устойчивости равновесных форм нити в линейном параллельном поле сил”.