The Monty Hall problem (and alike) settled

by Semjon Adlaj

The "Monty Hall problem" is much ridiculously discussed but hardly once and for all ever settled. So assume that a "Grand Prize" is hidden behind one of three doors, which we shall call *the right door*, whereas either one of the remaining two doors (without a prize behind it) is *a wrong door*. We shall be given that a door has been picked and that another door turned out to be a wrong door, that is, no prize was revealed upon opening it.

We shall not be as silly as to begin arguing about possible answers before clarifying the question. Certainly, we need not assume that Monty was either obliged to open a wrong door, or that he was unbiased upon deciding to open it. So, assume that he needs not necessarily open a door at all, and that he, furthermore, certainly never opens the right door. Suppose that he opens the (other) wrong door with probability p, if a wrong door was initially chosen, and he opens a wrong door with probability q, if the right door was initially chosen. The (unconditional) probability that the right door was initially chosen is $1/3$, and the total probability that Monty will open a door at all is then $(2p+q)/3$. With these assumptions clearly stipulated we calculate the conditional probability that the right door was initially chosen, given that Monty had opened a wrong door, that is, a door other than the (right) door initially chosen. This (conditional) probability is $q/(2p+q)$. Thus, whoever argues that the chances are doubled by choosing the third door (that is neither the door initially chosen nor the door that Monty opened) does tacitly assume that Monty was either obliged to open a door or (more generally) that his decision of opening a (wrong) door was not influenced by the initial choice of the door, that is, $p = q$. Whoever argues that the chances are unchanged by choosing the third door does tacitly assume that the probability of opening a door is directly proportional to the number (one or two) of the remaining (aside from the door initially chosen) wrong doors, that is $2p = q$. The third possible answer that argues for sticking to the initial choice might, in particular, be justified by assuming that Monty is biased so he might open a door if and only if the right door was initially chosen, that is $p = 0$ and $q = 1$. In this latter case, opening a wrong door (by Monty) merely confirms that the right door was initially chosen.

The three possible answers thereby presented are irreconcilable. They, of course, are not answers to a single question but are answers to three distinct questions. Infinitely many versions of the Monty Hall problem are possible by specifying the probabilities p and q, but this problem is hardly worthy of attention when these probabilities are not clearly articulated nor even brought to consciousness!

A typical (meaningless) discussion of the Monty Hall problem was presented in Mlodinow's book "The Drunkard's Walk: How Randomness Rules Our Lives". Unsurprisingly, much of this book is devoted to "proving" that the probability that both siblings turn out to be girls, given that one is known to be a girl is not $1/2$ but $1/3$. There is not a natural way to salvage a corresponding question in order to fit the latter answer, especially after much prolonged discussion (in that book) eventually deteriorating to restoring the probability $1/2$ if the girl was known to be named "Florida". Yet, we might arrange for the answer $1/3$ by stipulating an artificial condition that we are not free to (randomly) count a girl who has a sister, before we "mix" her with her sister, in order to count both sisters only once even if we meet them separately on two distinct occasions![1] "However", a girl named "Florida" would not be confused with her sister whose name would then differ from hers, "since" (as we are told, in many more words, by Mlodinow) the name "Florida" is rare.[2] With these stipulations articulated and the missing assumptions amended, the answers $1/3$ and $1/2$ are respectively justified. Bravo Mr. Mlodinow! I only wonder what would have Feynman had to tell you had you ever presented such (mixed state) arguments to him?![3] Surely, Mr. Mlodinow, they could have (much) enriched his funniest jokes collection!

[1]Then, of course, with this (much) strange and awkward assumption being made, the probability of meeting a girl from a two-sibling family, with at least one girl, is $1/3$, indeed!

[2]But is it as rare as the rarity with which two girls are given the same name? And are these rarities independent of each other? Mr. Mlodinow does not seem to be concerned with these particular details nor is he aware that even the simplest "mathematical" problem needs to be well-posed before presented or (what is worse) so "solved"!

[3]Mlodinow alleged to knew and be influenced by Feynman, and to be "teaching since 2005" at Caltech, although he is not currently (as of 2019 A.D.) enlisted (among the employees) on that institution's website.